Inspiring Greatness
INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Calculus AB

Volume: The Washer Method

April 23, 2020

INDEPENDENCE SCHOOL DISTRICT

Calculus AB

Lesson: April 23, 2020

Objective/Learning Target:

Students will calculate the volume of a solid of revolution using the washer method.

Warm-Up:

Watch Videos: Leading up to the Washer Method
The Washer Method

Read Article: Washer Method (scroll down to the section titled Washer Method)

Notes:

The Washer Method

Let f and g be continuous and nonnegative on the closed interval $[a, b]$, as shown in Figure 5.28(a). If $g(x) \leq f(x)$ for all x in the interval, then the volume of the solid formed by revolving the region bounded by the graphs of f and g $(a \leq x \leq b)$ about the x-axis is

$$\text{Volume} = \pi \int_a^b \{[f(x)]^2 - [g(x)]^2\}\, dx.$$

$f(x)$ is the **outer radius** and $g(x)$ is the **inner radius.**

Examples:

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$f(x) = \sqrt{25 - x^2} \text{ and } g(x) = 3$$

about the x-axis (see Figure 5.29).

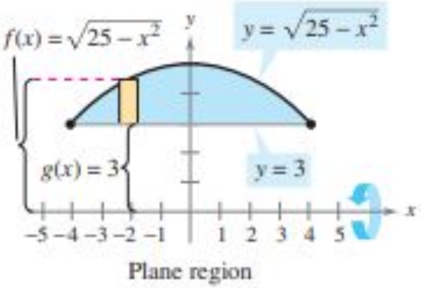
SOLUTION First find the points of intersection of f and g by setting $f(x)$ equal to $g(x)$ and solving for x.

$f(x) = g(x)$	Set $f(x)$ equal to $g(x)$.
$\sqrt{25 - x^2} = 3$	Substitute for $f(x)$ and $g(x)$.
$25 - x^2 = 9$	Square each side.
$16 = x^2$	
$\pm 4 = x$	Solve for x.

Using $f(x)$ as the outer radius and $g(x)$ as the inner radius, you can find the volume of the solid as shown.

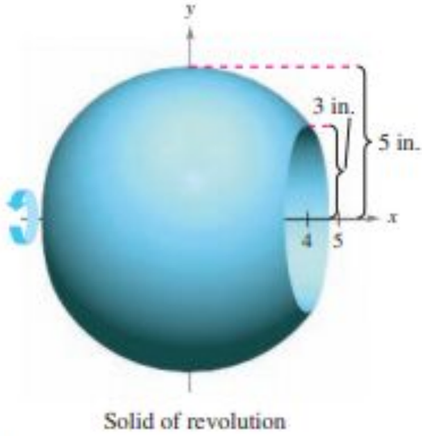
$\text{Volume} = \pi \int_{-4}^{4} \{[f(x)]^2 - [g(x)]^2\}\,dx$	Washer Method
$= \pi \int_{-4}^{4} \left[(\sqrt{25 - x^2})^2 - (3)^2\right] dx$	Substitute for $f(x)$ and $g(x)$.
$= \pi \int_{-4}^{4} (16 - x^2)\,dx$	Simplify.
$= \pi \left[16x - \frac{x^3}{3}\right]_{-4}^{4}$	Find antiderivative.
$= \frac{256\pi}{3}$	Apply Fundamental Theorem.
≈ 268.08	Round to two decimal places.

So, the volume of the solid is about 268.08 cubic inches.



(a)

FIGURE 5.29



(b)

Examples:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis, as shown in Figure 7.20.

Solution In Figure 7.20, you can see that the outer and inner radii are as follows.

$$R(x) = \sqrt{x} \qquad \text{Outer radius}$$

$$r(x) = x^2 \qquad \text{Inner radius}$$

Integrating between 0 and 1 produces

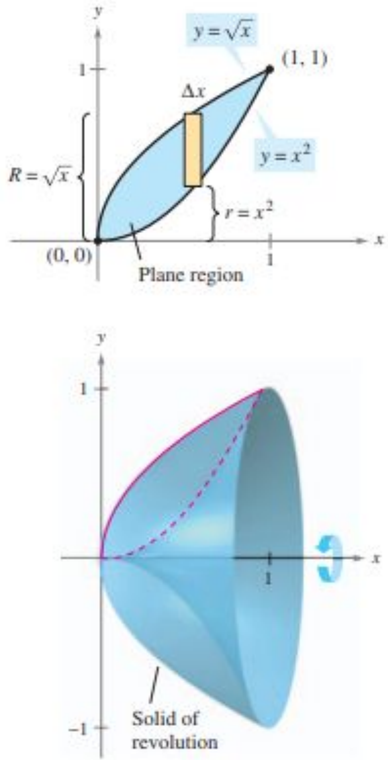
$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\,dx \qquad \text{Apply washer method.}$$

$$= \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$= \pi \int_0^1 (x - x^4)\,dx \qquad \text{Simplify.}$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 \qquad \text{Integrate.}$$

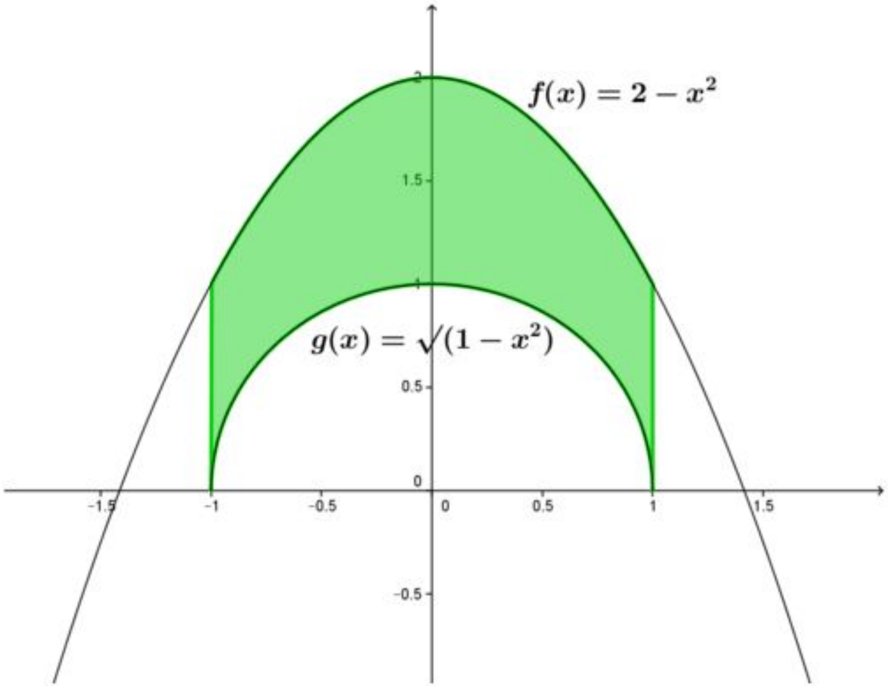
$$= \frac{3\pi}{10}.$$



Solid of revolution
Figure 7.20

Practice:

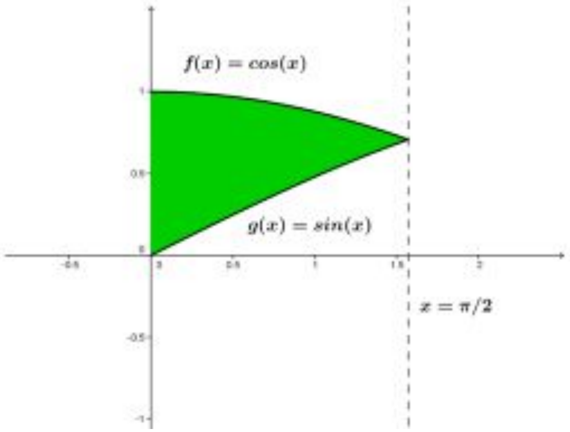
1) Find the volume of the solid generated by revolving the region bounded by $f(x) = 2 - x^2$, $g(x) = \sqrt{1 - x^2}$, $x = -1$, and $x = 1$ about the x-axis.



Practice:

2) Find the volume of the solid generated by revolving the region bounded by $y = \sin\frac{x}{2}$ and $y = \cos\frac{x}{2}$ on the interval $\left[0, \frac{\pi}{2}\right]$, about the x-axis.

The figure below shows the bounded region in Quadrant I that is to be revolved about the x-axis.



Answer Key:

Once you have completed the problems, check your answers here.

1) The volume of the solid is $\frac{22}{5}\pi$ cubic units.

2) The volume of the solid is π cubic units.

Additional Practice:

In your Calculus book read through Section 7.2 and complete problems 5, 13, and 31 on page 463

Interactive Practice

Extra Practice with Answers